Applications of q-Calculus

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1. Introduction and Definition

Let ∇ denote the set of all analytic function $\hat{h}(z)$ in the open unit disk $\ddot{O} = \{z \in \mathbb{C}: |z| < 1\}$ with normalized condition $\hat{h}(0) = 0$ and $\hat{h}'(0) = 1$, and series expansion of $\hat{h}(z) \in \nabla$ is given as

$$\hat{h}(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

The star-like and convex functions $\hat{h} \in \nabla$ of order α can be defined as

$$\mathcal{S}^*(\alpha) = \left\{\hat{h} \in \nabla: \mathrm{Re}\left(\frac{z\hat{h}'(z)}{\hat{h}(z)}\right) > \alpha, 0 \leq \alpha < 1, z \in \ddot{O}\right\},$$
$$\mathcal{C}(\alpha) = \left\{\hat{h} \in \nabla: \mathrm{Re}\left(\frac{\left(z\hat{h}'(z)\right)'}{\hat{h}'(z)}\right) > \alpha, 0 \leq \alpha < 1, z \in \ddot{O}\right\}.$$

Note that, if we take $\alpha = 0$, then the above defined classes of star-like and convex functions of order α convert into the well-known classes of star-like $(\mathcal{S}^*)$ and convex $(\mathcal{C})$ functions. In 1991 , the uniformly star-like functions (US) are introduced by Goodman [1] and defined as

$$\mathrm{US} = \left\{\hat{h} \in \nabla: \mathrm{Re}\left(\frac{z\hat{h}'(z)}{\hat{h}(z)}\right) > \left|\frac{z\hat{h}'(z)}{\hat{h}(z)} - 1\right|, z \in \ddot{O}\right\}.$$

$k - \mathrm{UST} = \{\hat{h} \in$

$k - \mathrm{UCV} = \{\hat{h} \in$

as follows:

$k^2((u-1)^2 + v^2)$

$w - 1 \,|$

Also, they defined these classes subject to the conic domain $\Omega_k (k \geq 0)$ (see [5]) as follows:

$$\begin{aligned} \Omega_k \quad &= u + iv: u^2 > k^2((u-1)^2 + v^2), \\ &= w: \mathrm{Re}\, w > k|w - 1| \end{aligned}$$

This domain represents the right-half plane for $k = 0$, hyperbola for $0 < k < 1$, parabola for $k = 1$, and ellipse $k > 1$. Furthermore, in [6], Shams et al. gave the generalization of k-uniformly star-like and k-uniformly convex functions and introduced a new subclasses $\mathrm{DD}(k, \gamma)$ and $\mathrm{SD}(k, \gamma)$ of analytic functions subject to the conic domain $\Omega_{k,\gamma} (k \geq 0)$, $0 \leq \gamma < 1$, as follows: After that, uniformly star-like functions (US) are studied by Ronning in [2] and Ma and Minda in [3].

Furthermore, in [4], Kanas and Wisniowska introduced the class k-uniformly star-like $(k - \mathrm{UST})$ and k-uniformly convex functions $(k - \mathrm{UCV}), k \geq 0$, as

$$\Omega_{k,\gamma} = \{u + iv: (u-\gamma)^2 > k^2((u-1)^2 + v^2)\}.$$

$\Omega_{k,\gamma}$.

The function $p_{k,\gamma}(z)$ is extremal function for domain

$$p_{k,\gamma}(z) = \begin{cases} \Psi_1(z) & \text{for } k = 0 \\ \Psi_2(z) & \text{for } k = 1 \\ \Psi_3(z) & \text{for } 0 < k < 1 \\ \Psi_4(z) & \text{for } k > 1 \end{cases}$$

where

$$\Psi_1(z) = \frac{1+z}{1-z}, \Psi_2(z) = 1 + \frac{2\gamma}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2$$
$$\Psi_3(z) = 1 + \frac{2\gamma}{1-k^2}\sinh^2\left\{\left(\frac{2}{\pi}\text{arccosk}\right)\text{arctanh}\sqrt{z}\right\}$$
$$\Psi_4(z) = 1 + \frac{\gamma}{k^2-1}\sin\left(\frac{\pi}{2K(i)}\int_0^{u(z)/\sqrt{t}}\frac{1}{\sqrt{1-x^2}\sqrt{1-(ix)^2}}dx\right) + \frac{\gamma}{1-k^2},$$
$$p_{k,\gamma}(z) = 1 + Q_1 z + Q_2 z^2 + \cdots,$$

and $i \in (0,1); k = \cosh \pi K'(i)/4K(i), K(i)$ is the first kind of Legendre's complete elliptic integral. For details, see [5]. where Indeed, from (9), we have

$$Q_1 = \begin{cases} \dfrac{2\gamma(2/\pi\arccos k)^2}{1-k^2} \text{ for } 0 \le \text{k} < 1 \\ \dfrac{8\gamma}{\pi^2} \text{ for k} = 1 \\ \dfrac{\pi^2\gamma}{4(1+t)\sqrt{t}K^2(t)(k^2-1)} \text{ for k} > 1 \end{cases}$$

$$Q_2 = \begin{cases} \dfrac{(2/\pi\arccos k)^2 + 2}{3} Q_1 \text{ for } 0 \le \text{k} < 1 \\ \dfrac{2}{3} Q_1 \text{ for k} = 1 \\ \dfrac{4K^2(t)(t^2+6t+1) - \pi^2}{24K^2(t)(1+t)\sqrt{t}} Q_1 \text{ for k} > 1 \end{cases}$$

as

The convolution of two functions $\hat{h}$ and g can be defined

$$(\hat{h} * g)z = z + \sum_{n=2}^{\infty} a_n b_n z^n$$

where $\hat{h}(z)$ is given by

(1) and $g(z)=z+\sum_{n=2}^{\infty}b_nz^n,(z\in \ddot{O})$

If $\hat{h}$ and $g\in\nabla$, we call that $\hat{h}$ is subordinate to g, written as $\hat{h}(z)\prec g(z)$, such that $\hat{h}(z)=g(u(z))$, where $u(z)\in\nabla$ is the Schwarz function in $\ddot{O}$ with conditions $u(0)=0$ and $|u(z)|<1$(see [7])

Let P represent Carathéodory class of $p\in\nabla$, in the open unit disk $\ddot{O}$, and given by

$$p(z)=1+\sum_{n=1}^{\infty}c_nz^n$$

such that

$$\operatorname{Re}(p(z))>0.$$

In geometric function theory (GFT), the quantum $(q-)$ calculus used as an important tools to study different families of analytic function and due to its application in mathematics and some related areas it has inspired the researcher very much. The quantum (or q^{-}) calculus is widely applied in various operators which include the q-difference (q-derivative) operator, and this operator plays an important role in GFT, quantum theory, number theory and statistical mechanics, etc. Jackson [8,9] was among the few researchers who defined the q-derivative and q-integral operator as well as provided some of their applications. Also, Ismail et al. [10] introduced research work in connection with function theory and q-theory. Later on, by using q-beta function, Gupta [11] introduced q-Baskakov-Durrmeyer operator, while q-Picard and q-Gauss-Weierstrass singular integral operators were introduced and studied by Aral, in [12]. Recently, Srivastava is one of the few researchers who studied univalent function theory by using a q-calculus (see, for details, [13,14]. Kanas and Raducanu [15] introduced Ruscheweyh q-differential operator, and Arif et al. [16] discussed some of its applications for multivalent functions. For more studies on q-analogous of operator, we refer $[15,17-19]$

$$[t]_q=1+q+q^2+\cdots+q^{n-1}\ (t=n\in\mathbb{N})$$
$$[n]_q!=\prod_{k=1}^{n}[k]_q,(n\in\mathbb{N})$$

where $[0]_q!=1$ and $[t]_{q]}=1-q^t/1-q,t\in\mathbb{C}$.

$$\begin{aligned}[t]_{n,q}&=\prod_{k=t}^{n-1}[k]_q,(n\in\mathbb{N})\\ [t]_q&=\frac{\Gamma_q(t+1)}{\Gamma_q(t)}\text{ and }\Gamma_{\mathrm{q}}(1)=1\end{aligned}$$

Definition 1 (see [6]). The q-number $[t]_q$ and q-factorial c for $q\in(0,1)$ is defined as

Definition 2. The q-generalized Pochhammer symbol and q-Gamma function be defined as

Definition 3 (see [9]). For $\hat{h}\in\nabla$, the q-derivative or (q -difference) operator be defined as

$$\partial_q \hat{h}(z) = \frac{-\hat{h}(z) + \hat{h}(qz)}{(q-1)z}, \mathrm{z} \in \ddot{\mathrm{O}}$$

Combining (1) and (21), we have

$$\partial_q \hat{h}(z) = 1 + \sum_{n=2}^{\infty} [n]_q a_n z^{n-1}$$

Note that

$$\partial_q z^n = [n]_q z^{n-1}, \partial_{\mathrm{q}} \left\{ \sum_{\mathrm{n}=1}^{\infty} \mathrm{a_n z^n} \right\} = \sum_{\mathrm{n}=1}^{\infty} [\mathrm{n}]_{\mathrm{q}} \mathrm{a_n z^{n-1}}$$

We can observe that

$$\lim_{q \to 1-} \partial_q \hat{h}(z) = \hat{h}'(z).$$

Definition 4 (see [10]). Let an analytic function $\hat{h} \in \delta_q^*$ if

$$\left| \frac{z\, \partial_q \hat{h}(z)}{\hat{h}(z)} - \frac{1}{1-q} \right| \leq \frac{1}{1-q}$$

$$\hat{h}(0) = \hat{h}'(0) = 1$$

Equivalently, we can rewrite (26) and (25) as follows (see [20]) :

$$\frac{z\, \partial_q \hat{h}(z)}{\hat{h}(z)} < \frac{1+z}{1-qz}.$$

In [21], Zhang et al. defined the class $k - \mathrm{P}_{q,\gamma}$ and generalized conic domain $\Omega_{k,q,\gamma}$.

Definition 5 (see [21]). Let $k \in [0,\infty)$, $q \in (0,1)$, and $\gamma \in \mathbb{C}/\{0\}$. A function $p(z) \in k - P_{q,\gamma}$ if and only if

$$p(z) < p_{k,\gamma,q}(z) = \frac{2p_{k,\gamma}(z)}{(1+q) + (1-q)p_{k,\gamma}(z)},$$

where $p_{k,\gamma}(z)$ is given by (9).

Geometrically, the function $p(z) \in k - \mathrm{P}_{q,\gamma}$ takes all values from the domain $\Omega_{k,q,\gamma}$ which is defined as follows:

$$\Omega_{k,q,\gamma} = \gamma \Omega_{k,q} + (1-\gamma)$$

where

$$\Omega_{k,q} = \left\{ w \colon \mathrm{Re} \left(\frac{(1+q)w}{(q-1)w+2} \right) > k \left| \frac{(1+q)w}{(q-1)w+2} - 1 \right| \right\}$$

Remark 1. When $q \longrightarrow 1-$, then $\Omega_{k,q,\gamma} = \Omega_{k,\gamma}$, where $\Omega_{k,\gamma}$ is considered by Shams et al. [6].

Remark 2. When $\gamma = 1$ and $q \longrightarrow 1-$, then $\Omega_{k,q,\gamma} = \Omega_k$, where Ω_k is the conic domain considered by Kanas and Wisniowska [4].

Remark 3. For $\gamma = 1$ and $q \longrightarrow 1-$, then $k - \mathrm{P}_{q,\gamma} = \mathrm{P}(p_k)$, where $\mathrm{P}(p_k)$ is the well-known class introduced by Kanas and Wisniowska [4].

Remark 4. For $\gamma = 1, k = 0$, and $q \longrightarrow 1-$, then $k - \mathrm{P}_{q,\gamma} = \mathrm{P}$, where P is the well-known Carathéodory class of analytic functions p

Mittag-Leffler introduced Mittag-Leffler function $\mathcal{H}_\alpha(z)$ in [22,23] as

$$\mathcal{H}_\alpha(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(\alpha n + 1)} z^n, (\alpha \in \mathbb{C}, \mathrm{Re}(\alpha)) > 0,$$

and its generalization $\mathcal{H}_{\alpha,\beta}(z)$ is introduced by Wiman [24] as

$$\mathcal{H}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(\alpha n + \beta)} z^n, (\alpha, \beta \in \mathbb{C}, \mathrm{Re}(\alpha), \mathrm{Re}(\beta)) > 0.$$

For more study about Mittag-Leffler functions (see $[25-30]$)

The q-Mittag-Leffler function defined by (see [31])

$$\mathcal{H}_{\alpha,\beta}(z, q) = \sum_{n=0}^{\infty} \frac{1}{\Gamma_q(\alpha n + \beta)} z^n, (\alpha, \beta \in \mathbb{C}, \mathrm{Re}(\alpha), \mathrm{Re}(\beta)) > 0$$

Note that q-Mittag-Leffler function is the specialized case of the q-Fox-Wright function $r\Phi_s(z, q)$ (see, for details, [32,33]). Since the q-Mittag-Leffler function $\mathcal{H}_{\alpha,\beta}(z, q)$ defined by (33) does not belong to the normalized analytic function class ∇.

Now, we define the normalization of this q-Mittag-Leffler function $\mathcal{R}_{\alpha,\beta}(z)$ as

$$\begin{aligned}
\mathcal{R}_{\alpha,\beta}(z, q) &= z\Gamma_q(\beta)\mathcal{H}_{\alpha,\beta}(z), \\
\mathcal{R}_{\alpha,\beta}(z, q) &= z + \sum_{n=2}^{\infty} \varphi_n(q, \beta) z^n, \\
\varphi_n(q, \beta) &= \frac{\Gamma_q(\beta)}{\Gamma_q(\alpha(n-1) + \beta)},
\end{aligned}$$

where $z \in \ddot{O}, \mathrm{Re}\,\alpha > 0$, and $\beta \in \mathbb{C}/\{0, -1, -2, \ldots\}$. Corresponding to $\mathcal{R}_{\alpha,\beta}(z, q)$ and for $\hat{h} \in \nabla$, we define the following q-differential operator $\mathcal{D}_q^m(\alpha, \beta)\colon \nabla \longrightarrow \nabla$ by

$$\left(\mathcal{D}_q^0(\alpha,\beta)\hat{h}\right)z = \hat{h}(z) * \mathcal{R}_{\alpha,\beta}(z,q)$$
$$\left(\mathcal{D}_q^1(\alpha,\beta)\hat{h}\right)z = z\mathcal{D}_q\left(\hat{h}(z) * \mathcal{R}_{\alpha,\beta}(z,q)\right)$$
$$\left(\mathcal{D}_q^2(\alpha,\beta)\hat{h}\right)z = \mathcal{D}_q\left(\mathcal{D}_q^1(\alpha,\beta)\hat{h}\right)z$$
$$\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z = \mathcal{D}_q\left(\mathcal{D}_q^{m-1}(\alpha,\beta)\hat{h}\right)z$$

We note that

$$\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z = z + \sum_{n=2}^{\infty} [n]_q^m \varphi_n(q,\beta) a_n z^n$$

Note that,

(i) For $\alpha = 0$ and $\beta = 1$, we have Salagean q-differential operator [34]

(ii) For $q \longrightarrow 1-, \alpha = 0$, and $\beta = 1$, we have Salagean differential operator [35]

(iii) For $m = 0$, we have $E_{\alpha,\beta}(z,q)$ (see [31])

(iv) For $m = 0$, we have $E_{\alpha,\beta}(z)$ (see [28])

We introduce a new subclass of analytic functions associated with newly defined q-differential operator (37).

Definition 6. Let $\hat{h}(z) \in \nabla$; then, $\hat{h}(z)$ is in the class $\mathcal{U}_{\alpha,\beta}^{q,m}(\gamma,k), \gamma \in \mathbb{C}/\{0\}$, if it satisfies the condition

$$\operatorname{Re}\left\{1 + \frac{1}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m((\alpha,\beta)\hat{h})\right)z} - 1\right)\right\} > k\left|\frac{1}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m((\alpha,\beta)\hat{h})\right)z} - 1\right)\right|,\ z \in \ddot{O}$$

or equivalently

$$\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(D_q^m(\alpha,\beta)\hat{h}\right)z} \in k - \mathrm{P}_{q,\gamma}.$$

Remark 5. For $\alpha = 0$ and $\beta = 1$, the class $\mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k) = k - \mathrm{US}\ (q,\gamma,m)$ studied by Hussain et al. [17].

Remark 6. For $m = 0,\ \alpha = 0,\ \beta = 1, q \longrightarrow 1-$, and $\gamma = 1/1-\eta, \eta \in \mathbb{C}/\{1\}$, the class $\mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k) = \mathrm{SD}(k,\eta)$ is studied by Shams et al. [6].

Remark 7. For $m = 0,\ \alpha = 0,\ \beta = 1, q \longrightarrow 1-$ and $\gamma = 2/1-\eta, \eta \in \mathbb{C}/\{1\}$, the class $\mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k) = \mathrm{DD}(k,\eta)$ is studied by Owa et al. [36].

Remark 8. For $k = 0, m = 0, \alpha = 0, \beta = 1, q \longrightarrow 1-$, and $\gamma = 1/1-\eta, \mathbb{C} \setminus \{1\}$, the class $\mathcal{U}\mathcal{S}_{\alpha,\beta}^{q,m}(\gamma,k) = \mathcal{S}^*(\eta)$ is the well-known subclass of star-like function.

Geometrically, an analytic function $\hat{h} \in \mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k)$ if and only if

$$\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}$$

take all values in the generalized conic domain $\Omega_{k,q,\gamma}$ given by (29). Taking geometrical interpretation into consideration, one can rephrase the above definition as follows.

Definition 7. Let an analytic function $\hat{h} \in \mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k)$ if and only if

$$\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} \prec p_{k,\gamma,q}(z),$$

where $p_{k,\gamma,q}(z)$ is given by (28).

2. Set of Lemmas

Lemma 1 (see [37]). If $F(z)$ is convex univalent in $\ddot{O}$ and let $p(z) = \sum_{n=1}^{\infty} p_n z^n \prec F(z) = \sum_{n=1}^{\infty} d_n z^n$ in $\ddot{O}$, then

$$|p_n| \leq |d_1|, \mathrm{n} \geq 1.$$

Lemma 2 (see [21]). Let $k \in [0,\infty)$ be fixed and

$$p_{k,\gamma,q}(z) = \frac{2p_{k,\gamma}(z)}{(1+q)+(1-q)p_{k,\gamma}(z)}.$$

Then,

$$p_{k,\gamma,q}(z) = 1 + \frac{2}{1+q}Q_1 z + \left\{\frac{2}{1+q}Q_2 - \frac{2(1-q)}{1+q}Q_1^2\right\}z^2 + \cdots,$$

where Q_1 and Q_2 are given by (12) and (13).

Lemma 3 (see [21]). Let $p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \in k - \mathrm{P}_{q,\gamma}$; then,

$$|p_n| \leq \frac{2}{1+q}|Q_1|, \mathrm{n} \geq 1.$$

Lemma 4 (see [38]). Let an analytic function $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \in \mathrm{P}$ and $\mathrm{Re}(p(z)) > 0$ for all z in $\ddot{O}$; then,

$$\left|c_2 - \mu c_1^2\right| \leq 2\,\mathrm{m}\ \{1, |2\mu - 1|\}, \forall \mu \in \mathbb{C}.$$

3. Main Results

Theorem 1. Let an analytic function $\hat{h}(z) \in \mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k)$; then,

$$\left(\mathcal{D}_q^m((\alpha,\beta)\hat{h})z\right) < z\exp\int_0^z \frac{p_{k,\gamma,q}(u(\xi))-1}{\zeta}d\xi,$$

where $u(z)$ is analytic in $\ddot{O}$ having condition $u(0)=0$ and $|u(z)|<1$. Furthermore, for $|z|=\rho$, we have

$$\begin{aligned}\exp\int_0^1 \frac{p_{k,\gamma,q}(-\rho)-1}{\rho}d\rho \quad &\leq \left|\frac{\left(D_q^m(\alpha,\beta)\hat{h}\right)z}{z}\right| \\ &\leq \exp\int_0^1 \frac{p_{k,\gamma,q}(\rho)-1}{\rho}d\rho,\end{aligned}$$

where $p_{k,\gamma,q}(z)$ is defined by (28).

Proof. If $\hat{h}(z)\in\mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma,k)$, then, from identity (41), we have

$$\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} = p_{k,\gamma,q}(u(z)),$$

$$\frac{\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} - \frac{1}{z} = \frac{p_{k,\gamma,q}(u(z))-1}{z}.$$

Integrating (50) and after some simplification, we have

$$\left(\mathcal{D}_q^m((\alpha,\beta)\hat{h})\right)z < z\exp\int_0^z \frac{p_{k,\gamma,q}(u(\xi))-1}{\zeta}d\xi.$$

Hence, (47) is proved. Noting that the univalent function $p_{k,\gamma}(z)$ maps the disk $|z|<\rho(0<\rho\leq 1)$ onto a region which is convex and symmetric with respect to the real axis, we see

$$\begin{aligned}p_{k,\gamma,q}(-\rho|z|) \quad &\leq \mathrm{Re}\left\{p_{k,\gamma,q}(u(\rho z))\right\} \\ &\leq p_{k,\gamma,q}(\rho|z|)(0<\rho\leq 1, \mathrm{z}\in \ddot{\mathrm{O}}).\end{aligned}$$

Using (51) and (52) gives

$$\int_0^1 \frac{p_{k,\gamma,q}(-\rho|z|)-1}{\rho}d\rho \leq \mathrm{Re}\int_0^1 \frac{p_{k,\gamma,q}(u(\rho(z)))-1}{\rho}d\rho \leq \int_0^1 \frac{p_{k,\gamma,q}(\rho|z|)-1}{\rho}d\rho,$$

for $z\in\ddot{O}$. Thus, inequality (51) leads us to

$$\int_0^1 \frac{p_{k,\gamma,q}(-\rho|z|)-1}{\rho}d\rho \leq \log\left|\frac{\left(D_q^m(\alpha,\beta)\hat{h}\right)z}{z}\right| \leq \int_0^1 \frac{p_{k,\gamma,q}(\rho|z|)-1}{\rho}d\rho,$$

$$p_{k,\gamma,q}(-\rho)\leq p_{k,\gamma}(-\rho|z|), p_{k,\gamma,q}(\rho|z|)\leq p_{k,\gamma,q}(\rho),$$

which implies that

$$\exp \int_0^1 \frac{p_{k,\gamma,q}(-\rho)-1}{\rho}\,d\rho \le \left|\frac{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{z}\right| \le \exp \int_0^1 \frac{p_{k,\gamma,q}(\rho)-1}{\rho}\,d\rho$$

Hence, the proof is complete.

For $\alpha = 0$ and $\beta = 1$, then we have the following known corollary proved by Hussain et al. in [17].

Corollary 1. Let an analytic function $\hat{h}(z) \in k$-US (q, γ, m); then,

$$\mathcal{D}_q^m \hat{h}(z) < z\exp \int_0^z \frac{p_{k,\gamma}(u(\xi))-1}{\zeta}\,d\xi,$$

where $u(z)$ is analytic in $\ddot{O}$ having condition $u(0) = 0$ and $|u(z)| < 1$. Furthermore, for $|z| = \rho$, we have

$\exp \int_0^1 \frac{p_{k,\gamma}(-\rho)-1}{\rho}\,d\rho \le \left|\frac{\mathcal{D}_q^m \hat{h}(z)}{z}\right| \le \exp \int_0^1 \frac{p_{k,\gamma}(\rho)-1}{\rho}\,d\rho$. where $p_{k,\gamma}(z)$ is defined by (9).

Theorem 2. If $\hat{h}(z) \in \mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma, k)$, then

$$|a_2| \le \frac{\delta}{[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)},$$

$$|a_n| \le \frac{\delta}{[n]_q^m\{[n]_q - 1\}\varphi_n(q,\beta)} \cdot \prod_{j=1}^{n-2}\left(1 + \frac{\delta}{[j+1]_q - 1}\right), \text{ for } n \ge 3,$$

where $\delta = 2|Q_1|/1 + q$ with Q_1 and $\varphi_n(q,\beta)$ is given by (12) and (35).

Proof. Let

$$\frac{z\,\partial_d\left(D_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} = p(z),$$

where $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ and $\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z$ is given by (37); then, (61) becomes

$$z + \sum_{n=2}^{\infty} [n]_q^{m+1}\varphi_n(q,\beta)a_n z^n = \left(\sum_{n=0}^{\infty} c_n z^n\right)\left(z + \sum_{n=2}^{\infty} [n]_q^m \varphi_n(q,\beta)a_n z^n\right)$$

Now, comparing the coefficients of z^n, we obtain

$$[n]_q^{m+1}\varphi_n(q,\beta)a_n = [n]_q^m\varphi_n(q,\beta)a_n + \sum_{j=1}^{n-1} [J]_q^m\varphi_j(q,\beta)a_jc_{n-j}$$

which implies

$$a_n = \frac{1}{[n]_q^m\{[n]_q - 1\}\varphi_n(q,\beta)}\sum_{j=1}^{n-1} [j]_q^m\varphi_j(q,\beta)a_jc_{n-j}$$

Using Lemma 4, we have $|a_n| \leq \frac{2|Q_1|}{(1+q)[n]_q^m\{[n]_q-1\}\varphi_n(q,\beta)}\sum_{j=1}^{n-1}[j]_q^m\varphi_j(q,\beta)\left|a_j\right|$

Let us take $\delta = 2|Q_1|/1+q$. Then, we have

$|a_n| \leq \dfrac{\delta}{[n]_q^m\{[n]_q - 1\}\varphi_n(q,\beta)}\sum_{j=1}^{n-1}[j]_q^m\varphi_j(q,\beta)\left|a_j\right|$

For $n = 2$, in (66), we have

$$|a_2| \leq \frac{\delta}{[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)}$$

which shows that (60) holds for $n = 2$. To prove (60), we use the principle of mathematical induction; for this, consider the case $n = 3$:

$$|a_3| \leq \frac{\delta}{[3]_q^m\{[3]_q - 1\}\varphi_3(q,\beta)}\{1 + [2]_q^m\varphi_2(q,\beta)|a_2|\}$$

Using (67), we have

$$|a_3| \leq \frac{\delta}{[3]_q^m\{[3]_q - 1\}\varphi_3(q,\beta)}\left\{1 + \frac{\delta}{[2]_q - 1}\right\}$$

which shows that (60) holds for $n = 3$. Let us assume that (60) is true for $n \leq t$, that is,

$$|a_t| \leq \frac{\delta}{[t]_q^m\{[t]_q - 1\}\varphi_t(q,\beta)}\prod_{j=1}^{t-2}\left(1 + \frac{\delta}{[j+1]_q - 1}\right), \text{ for } n = 3,4, \ldots$$

Consider

$$
\begin{aligned}
|a_{t+1}| &\le \frac{\delta}{[t+1]_q^m\{[t+1]_q-1\}\varphi_{t+1}(q,\beta)} \\
&\le \frac{\delta}{[t+1]_q^m\{[t+1]_q-1\}\varphi_{t+1}(q,\beta)} \\
&\times \left\{\begin{matrix} 1+[2]_q^m\varphi_2(q,\beta)|a_2|+[3]_q^m\varphi_3(q,\beta)|a_3| \\ +[4]_q^m\varphi_4(q,\beta)|a_4|+\cdots+[t]_q^m\varphi_t(q,\beta)|a_t| \end{matrix}\right\} \\
&\le \frac{\delta}{[t+1]_q^m\{[t+1]_q-1\}\varphi_{t+1}(q,\beta)} \\
&\times \left\{\begin{matrix} 1+\dfrac{\delta}{[2]_q-1}+\dfrac{\delta}{[3]_q-1}\left(1+\dfrac{\delta}{[2]_q-1}\right) \\ +\cdots+\dfrac{\delta}{[t]_q-1}\displaystyle\prod_{j=1}^{t-2}\left(1+\dfrac{\delta}{[j+1]_q-1}\right) \end{matrix}\right\} \\
&= \frac{\delta}{[t+1]_q^m\{[t+1]_q-1\}\varphi_{t+1}(q,\beta)}\prod_{j=1}^{t-1}\left(1+\frac{\delta}{[j+1]_q-1}\right)
\end{aligned}
$$

which shows that (60) holds for $n=t+1$. Hence, theorem is completed.

For $\alpha=0$ and $\beta=1$, then we have the following known corollary proved by Hussain et al., in [17].

Corollary 2. If $\hat{h}(z)\in\mathcal{U}_{\alpha,\beta}^{q,m}(\gamma,k)$, then

$$|a_2|\le\frac{\delta}{[2]_q^m\{[2]_q-1\}\varphi_2}$$

$$|a_n|\le\frac{\delta}{[n]_q^m\{[n]_q-1\}}\prod_{j=1}^{n-2}\left(1+\frac{\delta}{[j+1]_q-1}\right),\ \text{for } n\ge 3$$

where $\delta=2|Q_1|/1+q$ and Q_1 is given by (12).

Theorem 3. Let $\hat{h}(z)\in\mathcal{U}_{\alpha,\beta}^{q,m}(\gamma,k)$ of form (1) and $0\le k<\infty$ be fixed and $\mu\in\mathbb{C}$; then,

$$\left|a_3-\mu a_2^2\right|\le\frac{Q_1}{[3]_q^m\varphi_3(q,\beta)(1+q)\{[3]_q-1\}}\mathrm{m}\ \ [1,|2v-1|],$$

where

$$v=\frac{1}{2}\left(\begin{matrix}1-\dfrac{Q_2}{Q_1}+(1-q)Q_1-\dfrac{2Q_1}{(1+q)\{[2]_q-1\}} \\ +\mu\dfrac{Q_1[3]_q^m\{[3]_q-1\}}{\varphi_2(q,\beta)(1+q)\left([2]_q^m\{[2]_q-1\}\right)^2}\end{matrix}\right)$$

where Q_1 and Q_2 are given by (12) and (13).

Proof. Let $\hat{h}(z) \in \mathcal{U}\delta_{\alpha,\beta}^{q,m}(\gamma, k)$; then, from (12), we have

$$\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} = p_{k,\gamma}(u(z))z \in \text{Ö}$$

Let

$$p(z) = \frac{1}{(1-u(z))(1+u(z))^{-1}} = 1 + c_1 z + c_2 z^2 + \cdots$$

and this gives

$$u(z) = \frac{c_1}{2}z + \frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right)z^2 + \cdots$$

$$\begin{aligned} & p_{k,\gamma,q}(u(z)) \\ = \quad & 1 + \frac{Q_1 c_1}{(1+q)}z + \frac{1}{(1+q)}\left\{\frac{Q_2 c_1^2}{2} + \left(c_2 - \frac{c_1^2}{2}\right)Q_1 \right. \\ & \left. - \frac{(1-q)Q_1^2 c_1^2}{2}\right\}z^2 + \cdots, \end{aligned}$$

$$\begin{aligned} & \frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(D_q^m(\alpha,\beta)\hat{h}\right)z} \\ = \quad & 1 + [2]_q^m \varphi_2(q,\beta)\left\{[2]_q - 1\right\}a_2 z \\ & + \left\{[3]_q^m \varphi_3(q,\beta)\left\{[3]_q - 1\right\}a_3 \right. \\ & \left. - \left([2]_q^m \varphi_2(q,\beta)\right)^2\left\{[2]_q - 1\right\}a_2^2\right\}z^2. \end{aligned}$$

Using (79) in (76) and comparing with (80), we obtain

Theorem 4. If an analytic function $\hat{h}(z)$ ofform (1) satisfies the condition,

$$\sum_{n=2}^{\infty}\left\{\left\{[n]_q - 1\right\}(k+1) + |\gamma|\right\}|\varphi_n(q,\beta)|\left|[n]_q^m\right||a_n| \leq |\gamma|$$

then $\hat{h}(z) \in \mathcal{U}_{\alpha,\beta}^{q,m}(\gamma, k)$

Proof. Let we note that

$$\begin{aligned} \left|\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z} - 1\right| \quad & = \left| \frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z - \left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}\right. \\ & = \frac{\sum_{n=2}^{\infty}\ [n]_q^m \varphi_n(q,\beta)\left\{[n]_q - 1\right\}a_n z^n}{z + \sum_{n=2}^{\infty}\ [n]_q^m \varphi_n(q,\beta)a_n z^n} \\ & \leq \frac{\sum_{n=2}^{\infty}\ \left|[n]_q^m \varphi_n(q,\beta)\left\{[n]_q - 1\right\}\right||a_n|}{1 - \sum_{n=2}^{\infty}\ \left|[n]_q^m\right||\varphi_n(q,\beta)\ \|\ a_n|}. \end{aligned}$$

From (85), it follows that

$$1-\sum_{n=2}^{\infty}\left|[n]_q^m\right|\left|\varphi_n(q,\beta)\right|\left|a_n\right|>0.$$

To show that $\hat{h}(z)\in\mathcal{US}_{\alpha,\beta}^{q,m}(\gamma,k)$, it is suffices that

$$a_2=\frac{Q_1c_1}{[2]_q^m\varphi_2(q,\beta)(1+q)\left\{[2]_q-1\right\}}$$

$$a_3=\frac{1}{[3]_q^m\varphi_3(q,\beta)\left\{[3]_q-1\right\}}\times\left\{\begin{array}{c}Q_1c_2/1+q\\ +\frac{1}{1+q}\left(\frac{Q_2}{2}-\frac{Q_1}{2}-\frac{(1-q)Q_1^2}{2}+\frac{Q_1^2}{(1+q)\left\{[2]_q-1\right\}}\right)c_1^2\end{array}\right\},$$

$$a_3-\mu a_2^2=\frac{Q_1}{[3]_q^m\varphi_3(q,\beta)\left\{[3]_q-1\right\}(1+q)}\times\left\{c_2-\frac{1}{2}\left(\begin{array}{c}1-\frac{Q_2}{Q_1}+(1-q)Q_1-\frac{2Q_1}{(1+q)\left\{[2]_q-1\right\}}\\ +\mu\frac{Q_1[3]_q^m\left\{[3]_q-1\right\}}{\varphi_3(q,\beta)(1+q)\left([2]_q^m\left\{[2]_q-1\right\}\right)^2}\end{array}\right)c_1^2\right\}.$$

For any complex number μ and after some calculation, we have

$$a_3-\mu a_2^2=\frac{Q_1}{[3]_q^m\varphi_3(q,\beta)(1+q)\left\{[3]_q-1\right\}}\left\{c_2-vc_1^2\right\}$$

where v is given by (75). Using Lemma 4 on (82), we have the required results.

For $\alpha=0$ and $\beta=1$, then we have the following known corollary proved by Hussain et al. in [17].

Corollary 3. Let $\hat{h}(z)\in U_{\delta_{\alpha,\beta}^{q,m}}^{q,}(\gamma,k)$ of form (1) and $0\leq k<\infty$ be fixed and $\mu\in\mathbb{C}$; then,

$$\left|a_3-\mu a_2^2\right|\leq\frac{Q_1}{2[3]_q^m\left\{[3]_q-1\right\}}\text{m}\ \ [1,|2v-1|]$$

where

$$=\frac{1}{2}\left(\begin{array}{c}1-\frac{Q_2}{Q_1}+(1-q)Q_1-\frac{Q_1}{\left\{[2]_q-1\right\}}\\ +\mu\frac{Q_1[3]_q^m\left\{[3]_q-1\right\}}{2\left([2]_q^m\left\{[2]_q-1\right\}\right)^2}\end{array}\right)$$

where Q_1 and Q_2 are given by (12) and (13), where $p_{k,\gamma}(z)$ is defined by (9)).

$$\left|\frac{k}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right)\right|-\mathrm{Re}\left\{\frac{1}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right)\right\}\leq 1$$

From (86), we have

$$\begin{aligned}
&\leq\left|\frac{k}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right)\right|-\mathrm{Re}\left\{\frac{1}{\gamma}\left(\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right.\right.\\
&\leq\frac{k}{|\gamma|}\left|\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right|+\frac{1}{|\gamma|}\left|\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right|\\
&\leq\frac{(k+1)}{|\gamma|}\left|\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}-1\right|\\
&=\frac{(k+1)}{|\gamma|}\left|\frac{z\,\partial_q\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z-\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}{\left(\mathcal{D}_q^m(\alpha,\beta)\hat{h}\right)z}\right|\\
&\leq\frac{(k+1)}{|\gamma|}\left\{\frac{\left|\sum_{n=2}^{\infty}[n]_q^m\varphi_n(q,\beta)\left\{[n]_q-1\right\}\right||a_n|}{1-\sum_{n=2}^{\infty}\left|[n]_q^m\right||\varphi_n(q,\beta)||a_n|}\right\}1,
\end{aligned}$$

from (85)

When $q\longrightarrow 1-, m=0, \alpha=0, \beta=1$, and $\gamma=1-\eta$, with $0\leq\eta<1$, then we have the following.

Corollary 4 (see [6]). An analytic function $\hat{h}$ of form (1) belongs to the class $k-\mathrm{US}(1-2\eta)$ if it holds the following condition:

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\eta)\}|a_n|\leq 1-\eta$$

where $0\leq\eta<1$ and $k\geq 0$.

When $q\longrightarrow 1-, m=0, \alpha=0, \beta=1$, and $\gamma=1-\eta$, with $0\leq\eta<1$ and $k=0$, then we have the following.

Corollary 5 (see [39]). An analytic function $\hat{h}\in\mathrm{US}(1-\eta)$ of form (1) it holds

$$\sum_{n=2}^{\infty}\{n-\eta\}|a_n|\leq 1-\delta, 0\leq\eta<1.$$

Theorem 5. Let $\hat{h}(z)\in\mathcal{U}_{\alpha,\beta}^{q,m}(\gamma,k)$; then, $\hat{h}(\ddot{O})$ contains an open disk of radius:

$$\frac{[2]^m\varphi_2(q,\beta)\left\{[2]_q-1\right\}}{\left(2[2]_q^m\left\{[2]_q-1\right\}\varphi_2(q,\beta)\right)+\delta}$$

where $\delta = 2|Q_1|/1+q$ with Q_1 is given by (12). Proof. Let $u_0 \neq 0$ be a complex number such that $\hat{h}(z) \neq u_0$ for $z \in \ddot{O}$. Then,

$$\hat{h}_1(z) = \frac{u_0 \hat{h}(z)}{u_0 - \hat{h}(z)} = z + \left(a_2 + \frac{1}{u_0}\right) z^2 + \cdots$$

Since $\hat{h}_1(z)$ is univalent, so

$$\left|a_2 + \frac{1}{u_0}\right| \leq 2.$$

Now, using (59), we have

$$\left|\frac{1}{u_0}\right| \leq \frac{\left(2[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)\right) + \delta}{[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)}$$

Hence, we have

$$|u_0| \geq \frac{[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)}{\left(2[2]_q^m\{[2]_q - 1\}\varphi_2(q,\beta)\right) + \delta}$$

For $\alpha = 0$ and $\beta = 1$, then we have the following known corollary proved by Hussain et al. in [17].

Corollary 6. Let $\hat{h}(z) \in \mathcal{US}_{\alpha,\beta}^{q,m}(\gamma, k)$; then, $\hat{h}(\ddot{O})$ contains an open disk of radius:

$$\frac{[2]^m\{[2]_q - 1\}}{2[2]_q^m\{[2]_q - 1\} + \delta}$$

where $p_{k,\gamma}(z)$ is defined by (9) and $\delta = |Q_1|$ with Q_1 is given by (12).

4. Conclusion

Here, we have introduced and defined new subclass of analytic functions in generalized conic domain by using newly defined q-differential operator. This newly defined operator for analytic functions is the extension of q-Salagean operator. We investigated structural formula, coefficient estimates, and Fekete-Szegö problem, and also, we have successfully derived subordination result.

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